CHAPTER: 5
Gravitation

Law of Gravitation:-

Statement:-

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Explanation:-

Consider two bodies of masses ‘m_1’ and ‘m_2’ having distance ‘d’ between their centers. If ‘f’ is the force of attraction between these bodies then according to law of gravitation

$$F \propto m_1n_2 \longrightarrow (1)$$

$$F \propto \frac{1}{d^2} \longrightarrow (2)$$

From (1) and (2)

$$F \quad \frac{m_1m_2}{d^2}$$

$$F = (\text{constant}) \frac{m_1m_2}{d^2}$$

Here

Constant = G

So

$$F = \frac{Gm_1m_2}{d^2}$$

Here ‘G’ is constant of proportionality known as universal constant of gravitation its value is $G=6.67 \times 10 \frac{Nm^2}{Kg^2}$.

This value of G is very small therefore we do not feel this force of attraction between two bodies in our daily life.

Mass of the Earth:-

Consider a body of mass 'm' is placed on the surface of earth. The mass of earth is Me and the centre of earth is equal to radius of the earth.

According to law of gravitation.

$$F = \frac{G\, m_1 m_2}{d^2}$$

Here

$$g = 10\,m/s2$$

$$G = 6.67 \times 10^{-11}\ \frac{Nm^2}{Kg^2}.$$

$$R = 6.4 \times 10^6 m$$

Now

$$Me = \frac{gR^2}{G}$$

Putting values

$$Me = \frac{(10)\,(6.4 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$Me = 6 \times 10^{24}\ kg$$

$$\boxed{\mathbf{Me = 6 \times 10^{24}\ kg}}$$

Law of Gravitation and Newtons third Law of Motion:-

According to newtons law of gravitation the body of mass m_1 attracts the body of mass m_2 attracts the body of mass m_1 with a force. These two forces are equal in a magnitude but opposite in direction. Hence we can say that law of gravitation and newtons third law are similar.

Variation of 'g' with Altitude:-

We know that the value of acceleration due to gravity is given by

$$G = \frac{GMe}{R^2} \longrightarrow (1)$$

The relation shows that the value of ‘g’ is inversely proportional to square of the distance from centre of earth. It means that if the distance from centre of earth increases then value of ‘g’ decreases.

If a body of mass ‘m’ is placed at an altitude (Height) ‘h’ from centre of earth then

Total distance from centre of earth = R + h

Now

Equation no (1) becomes

$$G = \frac{GMe}{R^2}$$

Put $R = R + h$

$$G = \frac{GMe}{(4R)^2}$$

This is the value of ‘g’ at an altitude ‘h’.

Question

What is the value of ‘g’ at a height equal to one earth radius?

Ans. We know that the value of g at altitude ‘h’ is given by

$$g = \frac{GMe}{(R + h)^2} \longrightarrow (1)$$

The height is equal to one earth radius then h = R put in (1)

$$g = \frac{GMe}{(2R)^2}$$

$$g = \frac{GMe}{2R^2}$$

$$g = \frac{GMe}{4R^2}$$

$$G = \frac{1}{4} \frac{GMe}{R^2}$$

So if height of a body is equal to one earth radius then value of ‘g’ becomes one fourth of its value on earth.

Question

What is the value of 'g' at a height equal to two earth radius?

Ans. We know that the value of g at altitude 'h' is given by

$$g = \frac{GMe}{(R+h)^2} \longrightarrow \quad (1)$$

The height is equal to one earth radius then h = 2R put in (1)

$$g = \frac{GMe}{(R+2R)^2}$$

$$g = \frac{GMe}{(3R)^2}$$

$$g = \frac{GMe}{9R^2}$$

$$G = \frac{1}{9} \frac{GMe}{R^2}$$

So if height of a body is equal to two earth radius then value of 'g' becomes one ninth of its value on earth.

Satellite:-

An object that revolves around a planet is called satellite.

Example:-

The moon revolves around the earth so moon is the natural satellite of earth.

Artificial Satellite:-

Artificial satellite is an object that is sent into space and move around the earth under the action of gravity. Artificial satellite are used for communication and space research.

Geo Stationary Satellite:-

Those satellite that take 24 hours to complete one rotation about their axis are called geostationary satellites remain stationary with respect to earth and orbit of these satellite is called geostationary orbit.

Motion of Artificial Satellites:-

Artificial satellites are the objects that are sent into space and move under the action of gravity.

Explanation:-

A satellite requires centripetal force to move around the earth. This centripetal force is provided by the gravitational force of attraction between the satellite and the earth consider a satellite of mass 'm' revolving around the earth at a height 'h' with velocity 'v'. The radius of the orbit of satellite is 'r'. The centripetal force is given by

$$Fc = \frac{mv2}{r}$$

This centripetal force is provided by the gravitational force of attraction. In this case the gravitational force of attraction is equal to weight of the body

$$F_g = \text{weight}$$

$$F_g = mg$$

Here

Centripetal force = Gravitational force

$$F_c = F_g$$

$$\frac{mv^2}{r} = mg$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$\frac{v^2}{r} = g$$

$$\sqrt{v^2} = \sqrt{gr}$$

$$v^2 = gr$$

$$v = gr \qquad (1)$$

When satellite is moving at a height 'h' from the surface of earth then

$$R = r + h \text{ put in } \quad (1)$$

$$V = \sqrt{g\,(R+h)}$$

But if the satellite is revolving close to the earth

Then

$$V = \sqrt{gr}$$

Here $g = 10$ m/sec2

$R = 6.4 \times 106$m

Putting values

$$V = \sqrt{gR}$$

$$V = (10)\,(6.4 \times 106)$$

$$V = 8000 \text{ m/sec}$$

Or

$$V = 8 \text{ km/sec.}$$